7 Electric and magnetic fields in media

1. Derive Gauss's law for dielectrics,

$$\int_{\partial V} \vec{ds} \cdot \vec{D} = Q_{\text{ext}},$$

where Q_{ext} is the total external charge contained in the volume V.

2. Calculate the electric field and the electric potential of a charge q placed in the middle of a dielectric sphere of radius a and permittivity ϵ. Compute the discontinuities and continuities of $\vec{E}$ and $\vec{D}$ across the boundary of the sphere.

3. Consider an infinite dielectric composed of two media with permittivities ϵ_1 and ϵ_2, separated by a plane. Find the electric field and potential of a point charge q placed in the medium ϵ_1 at a distance d from the plane. Calculate the density of polarization charges on the plane.

4. Consider a dielectric sphere with uniform polarization $\vec{P}$. It can be represented as two positively and negatively charged spheres slightly displaced with respect to one another. Using this representation, calculate the surface charge density produced by the polarization. Compare to that given by the relation

$$\sigma_P = \vec{P} \cdot \vec{n},$$

where $\vec{n}$ is the unit vector normal to the sphere.

5. Calculate the potential of a dielectric sphere of radius a, polarized uniformly in the direction z with the magnitude P.

6. Consider a dielectric sphere of radius a and permittivity ϵ embedded in a uniform electric field of magnitude E_0. There are no free charges. Find the electric field both inside and outside of the sphere. Calculate the polarization vector $\vec{P}$. Find the charge density induced on the boundary of the sphere.

7. Consider an infinite dielectric of permittivity ϵ filled with a uniform electric field of magnitude E_0. Find the electric field produced by a spherical cavity of radius a cut in this dielectric. Find the polarization vector $\vec{P}$. Calculate the charge density induced at the boundary of the sphere.

8. Find the magnetic field $\vec{H}$ and magnetic induction $\vec{B}$ of a uniformly magnetized sphere of radius a with magnetization $\vec{M}$ parallel to the z-axis.

9. Consider a finite cylinder of length L and radius a, placed parallel to the z-axis and with uniform magnetization $\vec{M} = M\vec{e}_z$. Calculate the magnetic field $\vec{H}$ everywhere on the z-axis. Consider the limit $L \to \infty$ (with a fixed) to find the magnetic field $\vec{H}$, the magnetic induction $\vec{B}$ and the vector potential $\vec{A}$ everywhere in space. What is the value of the magnetization current $\vec{J}_M$?

10. A sphere of radius a and magnetic permittivity μ is inserted into a uniform magnetic field $\vec{B}_0$. Calculate the magnetization of the sphere, the magnetization currents, the resulting magnetic field and magnetic induction everywhere in space.