Séance 3 : Corrigés

Exo 1

$$\begin{cases} P_\gamma^N = (E, E, 0, 0) & (\text{car } ds^2 = 0) \\ P_\gamma'^N = (E', P', 0, 0) \end{cases}$$

$$\begin{pmatrix} E' \\ P' \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \operatorname{ch}\theta & \operatorname{sh}\theta & 0 & 0 \\ \operatorname{sh}\theta & \operatorname{ch}\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ E \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} E' = e^\theta E \\ P' = e^\theta E = E' \end{cases} \Rightarrow \boxed{P_\gamma'^N = e^\theta P_\gamma^N}$$

Exo 2

$${}^2D + {}^2D \rightarrow {}^4He + \Delta E$$

$$\Rightarrow \Delta E = \Delta m \cdot c^2 = (2m_D - m_{He})c^2 = 0{,}0228 \text{ GeV}$$

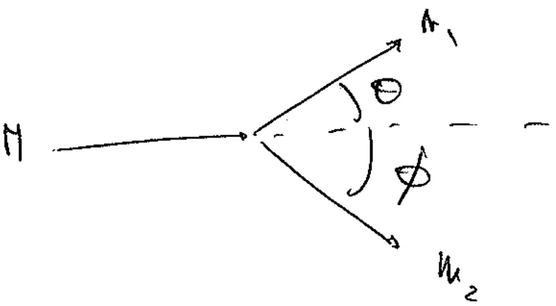
$$= 3{,}65 \cdot 10^{-12} \text{ J}$$

Or $\underset{1g}{M} c^2 = 9 \cdot 10^{13}$ J

$$\rightarrow \frac{Mc^2}{m_D c^2} = 3 \cdot 10^{23} \text{ atomes} = 1{,}5 \cdot 10^{23} \text{ réactions.}$$

$$\Delta E_{TOT} = \Delta E \cdot \#_{r} = 5{,}5 \cdot 10^{11} \text{ J} = \boxed{3{,}4 \cdot 10^{21} \text{ GeV}}$$

Exo 3



$$\begin{cases} p^\mu = (E, \vec{p}) \\ p_1^\mu = (E_1, \vec{p}_1) \\ p_2^\mu = (E_2, \vec{p}_2) \end{cases} \qquad \begin{cases} \vec{p} = (p, 0, 0) \\ \vec{p}_1 = (p_1 \cos\theta, p_1 \sin\theta, 0) \\ \vec{p}_2 = (p_2 \cos\phi, p_2 \sin\phi, 0) \end{cases}$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \Leftrightarrow \begin{cases} p = p_1 \cos\theta + p_2 \cos\phi \\ 0 = p_1 \sin\theta + p_2 \sin\phi \end{cases}$$

$$|\vec{p}|^2 = |\vec{p}_1 + \vec{p}_2|^2 = |\vec{p}_1|^2 + |\vec{p}_2|^2 + 2\vec{p}_1 \cdot \vec{p}_2$$

$$= p_1^2 + p_2^2 + 2p_1p_2(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$\Leftrightarrow \quad = p_1^2 + p_2^2 + 2p_2(p\cos\phi - p_2)$$

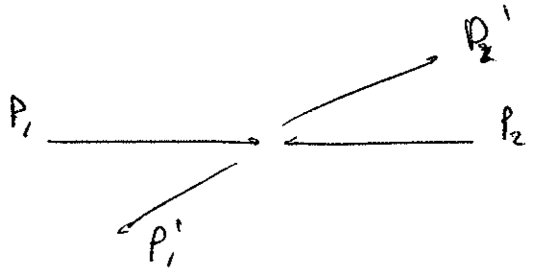
$$\Leftrightarrow \underbrace{E^2}_{\cancel{E_1^2} + \cancel{E_2^2} + 2E_1E_2} - M^2 = \cancel{E_1^2} - m_1^2 + \cancel{E_2^2} - m_2^2 + 2\sqrt{E_2^2 - m_2^2}\sqrt{E^2 - M^2}\cos\phi - 2\sqrt{E_2^2 - m_2^2}$$

$$\Rightarrow \boxed{\cos\phi = \frac{2E_2(E_1+E_2) + m_1^2 - m_2^2 - M^2}{2\sqrt{E_2^2 - m_2^2}\sqrt{(E_1+E_2)^2 - M^2}}}$$

Idem pour θ:

$$\boxed{\cos\theta = \frac{2E_1(E_1+E_2) - m_1^2 + m_2^2 - M^2}{2\sqrt{E_1^2 - m_1^2}\sqrt{(E_1+E_2)^2 - M^2}}}$$

Exo 4



$$\left\{\begin{array}{l} P_1^\mu = (E_1, \vec{P}_1) \\ P_2^\mu = (E_2, \vec{P}_2) \end{array}\right) \leadsto P_1^i + P_2^i \overset{SCM}{=} 0$$

$$\left.\begin{array}{l} P_1'^\mu = (E_1', \vec{P}_1') \\ P_2'^\mu = (E_2', \vec{P}_2') \end{array}\right) \leadsto P_1'^i + P_2'^i \overset{SCM}{=} 0 \qquad i = 1,2,3.$$

$$\Rightarrow \left\{\begin{array}{l} \vec{P}_1 = -\vec{P}_2 \\ \vec{P}_1' = -\vec{P}_2' \end{array}\right. \quad \text{et} \quad E_1 + E_2 = E_1' + E_2'$$

$$(P_1^\mu + P_2^\mu)^2 = (P_1'^\mu + P_2'^\mu)^2$$

$$E_1 E_2 - \vec{P}_1 \vec{P}_2 = E_1' E_2' - \vec{P}_1' \vec{P}_2'$$

$$\Leftrightarrow \sqrt{m_1^2 + p_1^2}\sqrt{m_2^2 + p_1^2} + p_1^2 = \sqrt{m_1^2 + p_1'^2}\sqrt{m_2^2 + p_1'^2} + p_1'^2$$

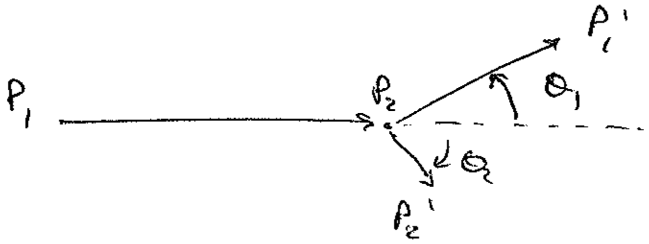
Soit $f(x) = \sqrt{m_1^2 + x^2}\sqrt{m_2^2 + x^2} + x^2$

$f(p_1) = f(p_1') \Rightarrow p_1 = p_1'$ ssi f strictement monotone (ie: $f'(x) > (<) \, 0$)

Or, $f'(x) > 0 \Rightarrow p_1 = p_1'$

$$\Rightarrow \boxed{p_1 = -p_2 = p_1' = -p_2'}$$

$\boxed{\text{Exo 5}}$



Cas non-relativiste:

Conservation de l'énergie cinétique:

$$\frac{\vec{P}_1^{\,2}}{2m_1} = \frac{\vec{P}_1'^{\,2}}{2m_1} + \frac{\vec{P}_2'^{\,2}}{2m_2}$$

Conservation de l'impulsion:

$$\vec{P}_2'^{\,2} = (\vec{P}_1 - \vec{P}_1')^2 = \vec{P}_1^{\,2} + \vec{P}_1'^{\,2} - 2|\vec{P}_1||\vec{P}_1'|\cos\theta_1$$

$$\Rightarrow \; P_1'^2\left(\frac{1}{2m_1} + \frac{1}{2m_2}\right) + P_1^2\left(\frac{1}{2m_2} - \frac{1}{2m_1}\right) - \frac{2P_1P_1'}{2m_2}\cos\theta = 0$$

$$\Delta = \frac{\cos^2\theta}{m_2^2} P_1'^2 - 4P_1'^2\left(\frac{1}{4m_2^2} - \frac{1}{4m_1^2}\right) \geq 0$$

$\Rightarrow$ Pour qu'$\exists$ une solution au moins.

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \; \boxed{-\frac{m_2}{m_1} \leq \sin\theta \leq \frac{m_2}{m_1}}$$

Cas relativiste:

$$\begin{cases} p_1^\mu = (E_1, P_1, 0, 0) \\ p_2^\mu = (m_2, 0, 0, 0) \\ p_1'^\mu = (E_1', P_1'\cos\theta_1, P_1'\sin\theta_1, 0) \\ p_2'^\mu = (E_2', P_2'\cos\theta_2, P_2'\sin\theta_2, 0) \end{cases}$$

Conservat° du 4-vecteur Énergie-impulsion :

$$\begin{cases} E_1 + m_2 = E_1' + E_2' \\ P_1 = P_1' \cos\theta_1 + P_2' \cos\theta_2 \\ 0 = P_1' \sin\theta_1 + P_2' \sin\theta_2 \end{cases}$$

$$(P_1^\mu - P_1'^\mu)^2 = (P_2'^\mu - P_2^\mu)^2$$

$$\rightarrow \sqrt{E_1^2 - m_1^2}\sqrt{E_1'^2 - m_1^2}\,\cos\theta = E_1 E_1' + m_2 E_1' - m_2 E_1 - m_1^2$$

Soient $\eta \equiv \frac{E_1}{m_1}$, $\eta' \equiv \frac{E_1'}{m_1}$ et $\mu \equiv \frac{m_2}{m_1}$

$$\rightarrow \cos\theta = \frac{\eta\eta' - 1 + \mu(\eta' - \eta)}{\sqrt{\eta^2 - 1}\sqrt{\eta'^2 - 1}}$$

Soient $\begin{cases} \xi \equiv \sqrt{\eta^2 - 1} = P_1/m_1 \\ \xi' \equiv \sqrt{\eta'^2 - 1} = P_1'/m_1 \end{cases}$ avec $\xi' \in [0 ; \xi]$

$$\rightarrow \boxed{\cos\theta = \frac{\sqrt{1+\xi^2}\sqrt{1+\xi'^2} + \mu\left(\sqrt{1+\xi'^2} - \sqrt{1+\xi^2}\right)}{\xi\xi'}}$$

$$\begin{cases} \xi \gg 1 \Rightarrow \text{Relativiste} \\ \xi \ll 1 \Rightarrow \text{Non-relativiste} \end{cases}$$

$\cos\theta$ possède-t-il un extremum ?

$$\frac{d}{d\,\xi'/\xi}\cos\theta = 0 \Rightarrow \left(\frac{\xi'}{\xi}\right)^* = \frac{\sqrt{1-\mu^2+\frac{1}{\xi^2}+\frac{2\mu\sqrt{1+\xi^2}}{\xi^2}}}{\mu\sqrt{1+\xi^2}}$$

$$\begin{cases} \lim\limits_{\xi\to 0}\left(\frac{\xi'}{\xi}\right)^* = \infty \\ \lim\limits_{\xi\to\infty}\left(\frac{\xi'}{\xi}\right)^* = 0 \end{cases}$$

Donc, dans la limite relativiste $\left(\frac{\xi'}{\xi}\right)^*$ constitue un extremum (minimum).

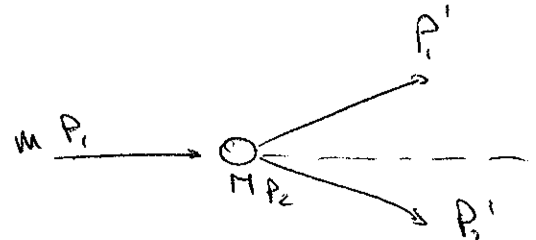
$\boxed{\text{Exo 6}}$

Dans le cas où $P_1 = E_1$ et $P_1' = E_1'$:

$$\cos\theta = 1 - m_2\,\frac{E_1 - E_1'}{E_1 E_1'}$$

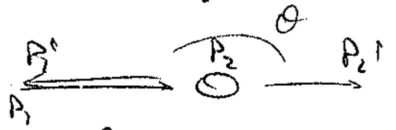
$\boxed{\text{Exo 7}}$

a) Classiquement :



$$\begin{cases} \dfrac{P^2}{2m} = \dfrac{P_1'^2}{2m} + \dfrac{P_2'^2}{2M} \\ \vec{P}^2 = \vec{P}_1'^2 + \vec{P}_2'^2 + \underbrace{2\vec{P}_1'\cdot\vec{P}_2'}_{= -2P_1P_2} \end{cases}$$

Si frontal :



$\cos\theta = -1$

$$\Rightarrow P_2' = 2 \cdot \frac{P_1'}{1 - m/M}$$

Soit $\Delta E_2 = \frac{P_2'^2}{2M}$ (gain par rapport au repos)

$$\frac{\Delta E_2}{E_{2ini}} = \frac{P_2'^2/2M}{P^2/2m} \underset{\mu = m/M}{=} \frac{4\mu}{(1+\mu)^2} \xrightarrow{\mu \to 0} 0 .$$

b/ Cas relativiste :

$$E_1' = \frac{E_1(m^2+M^2) + 2m^2M}{m^2+M^2+2E_1M}$$

$$\frac{\Delta E_2}{E_{ini}} = \frac{E_2' - M}{E_1} = \frac{E_1 - E_1'}{E_1} \xrightarrow{\mu \to 0} \frac{2E_1}{M+2E_1} \neq 0$$