3 Relativistic energy and momentum

Questions marked with an asterisk (*) are more difficult and not mandatory.

1. Photons are particles of zero mass. Find the transformation rule of the photon energy and momentum under a boost with the velocity V parallel to the photon momentum. Express the result in terms of the rapidity parameter.
2. The mass of the deuterium atom (D) is 1.8756 GeV (1GeV $= 1.602 \cdot 10^{-10}$J), while the mass of the helium atom (^{4}He) is 3.7284 GeV. Find the energy released in the conversion of 1 g of deuterium into helium (nuclear fusion).
3. A particle of mass M moving with the velocity V decays into two particles of masses m_1 and m_2. Find the relation between the energies of the resulting particles and the directions of their velocities.
4. Consider the elastic scattering of two particles of masses m_1 and m_2 from the center-of-momentum frame. Let $\vec{p}_1$, $\vec{p}_2$ be the three-momenta before the scattering and $\vec{p}_1'$, $\vec{p}_2'$ the three-momenta after the scattering process. Show that all momenta lie on a circle, i.e. are coplanar and of equal norm.
5. A moving particle of mass m_1 collides with the particle at rest of mass m_2. After the collision the velocity of the particle 1 changes the direction by the angle θ. Demonstrate that in the case $m_1 > m_2$ the angle θ cannot exceed some maximum value. Find this value. Do the same calculation in Galilean relativity and comment on the result.
6. Consider the collision of the preceding question when the incident particle is massless. Find the relation between the energy of the incident particle after collision and its scattering angle.
7. Consider the non-relativistic head-on collision of a light particle of mass m with a heavy one of mass M at rest. Show that the energy transferred to the heavy particle, measured in units of the energy of the incident particle, goes to zero in the limit $m/M \to 0$. Demonstrate that this is not the case in the relativistic case.
8. Protons present in cosmic rays may interact with the photons of the cosmic microwave background. Consider a head-on collision (zero impact parameter) of a proton with a photon of the energy 2.6×10^{-4}eV. What minimum energy of the proton is needed to produce a π^0-meson in the reaction $p + \gamma \to p + \pi^0$ (the masses of particles are $m_p c^2 = 0.938$GeV, $m_\pi c^2 = 0.135$GeV, $m_\gamma = 0$) ?

*9. A particle of mass M which is at rest decays into three particles with masses m_1, m_2 and m_3. Find the maximum energy which can be carried away by one of these particles.