2 Formalism of 4-vectors

1. Making use of the fact that the interval $s^2 = x_\mu x^\mu = (xx)$ is invariant under Lorentz transformations, find the matrix A by which the *covariant* vectors are transformed,

$$x_\mu = A_\mu{}^\nu x'_\nu.$$

Express A in terms of the Lorentz transformation matrix $\Lambda^\mu{}_\nu$. Find A explicitly in the case of the boost in the direction x with the rapidity θ.

2. Let x^μ and y^μ be two Lorentz vectors. Demonstrate that

$$x_\mu y^\mu = x^\mu y_\mu$$

3. Let x^μ be a time-like vector. Find a Lorentz transformation such that in the new frame the space components of the vector x^μ are zero [1].
4. Let x^μ be a space-like vector. Find a Lorentz transformation such that in the new frame the time component of the vector x^μ is zero.
5. Let $x^\mu_{(1)}$ be a time-like vector. Construct three vectors such that the resulting four vectors are linearly-independent and mutually orthogonal in the usual sense $(x_{(i)}x_{(j)}) = 0$ at $i \neq j$ where $i, j = 1, 2, 3, 4$. Are they time/space/light-like ? Is there a time-like vector orthogonal to $x^\mu_{(1)}$?
6. Let x^μ be a null vector. Is it possible to find three more linearly independent vectors satisfying the orthogonality properties of the preceding exercise ?
7. Let x^μ be a null vector. Find a linearly independent null vector.
8. Let x^μ and y^μ be a pair of linearly-independent null vectors. Construct two more linearly-independent vectors which are orthogonal to x^μ and y^μ and to each other.
9. Let $\Lambda^\mu{}_\nu$ be the matrix of the boost in the x-direction. Find the inverse matrix. Interpret the result.
10. Demonstrate that $\epsilon_{0123} = -1$.
11. Write out the differential operator $\partial^\mu \partial_\mu$ in components.
12. Find a Lorentz transformation that reverses time $(t' = -t)$ and leaves the spatial directions invariant. Does this transformation preserve $\epsilon^{\mu\nu\rho\sigma}$?

1. In this and a few subsequent exercises choose the orientation of the spatial axes so as to simplify calculations.