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PHYS-F202 Devoir 2

Séance 3 - exo 8:

we take $c=1$

$p+\gamma \to p+\pi^0$

$m_p = 0{,}938.10^9$ eV/c²

$m_{\pi^0} = 0{,}135.10^9$ eV/c²

$E_\gamma = 2{,}6.10^{-4}$ eV

Before : $\underset{p}{\longrightarrow}\ \underset{\gamma}{\longleftarrow}$

After : $\underset{p'}{\longleftarrow}\ \underset{\pi}{\longrightarrow}$

In term of four-vector : $P_i^\mu = \left(E_p + E_\gamma , \vec{P}_p - \vec{P}_\gamma\right)$

$P_f^\mu = \left(E_{p'} + E_\pi , \vec{P}_\pi - \vec{P}_{p'}\right)$

Conservation of four-momentum : $\boxed{P_i^\mu P_{i\mu} = P_f^\mu P_{f\mu}}$

$$\to \left(E_p + E_\gamma\right)^2 - \left(P_p - P_\gamma\right)^2 = \left(E_{p'} + E_\pi\right)^2 - \left(P_\pi - P_{p'}\right)^2$$

We choose the referential of center of momentum of π & p'.

To have E_p minimal, P_π & $P_{p'}$ must be equal to 0.

$$\Leftrightarrow m_p^2 + \cancel{P_p^2} + \cancel{E_\gamma^2} + 2E_\gamma\left(m_p^2 + P_p^2\right)^{1/2} - \cancel{P_p^2} - \cancel{P_\gamma^2} + 2P_pP_\gamma = \left(m_p + m_\pi\right)^2$$

we remember that $P_\gamma = E_\gamma$ because $m_\gamma = 0$

$$\Leftrightarrow \left(m_p^2 + P_p^2\right)^{1/2} + P_p = \underbrace{\left(\left(m_p + m_\pi\right)^2 - m_p^2\right)2E_\gamma^{-1}}_{\overset{\text{not}}{=}\ \Delta}$$

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$\Leftrightarrow (m_p^2 + P_p^2)^{\frac{1}{2}} + P_p = \Delta$

$\Leftrightarrow m^2_p + \cancel{P^2_p} = \Delta^2 + \cancel{P^2_p} - 2\Delta P_p$

$\Leftrightarrow P_p = \dfrac{m^2_p - \Delta^2}{-2\Delta} \overset{N.A.}{\simeq} 2{,}61.10^{20}\ eV/c^2$

$E_{p\ min} = (m_p^2 + P_p^2)^{\frac{1}{2}}$ $\boxed{E_{p\ min} \simeq 2{,}61.10^{20}\ eV}$

Séance 4 - exo 13:

1) $\underline{\text{in:}}$ $\underline{r<R}$ $\boxed{\vec{E} = \vec{0}}$ (because conducting)

$\underline{\text{out:}}$ $\underline{r \geq R}$ $\int_S \vec{E} \cdot d\vec{S} = \dfrac{Q_{int}}{\varepsilon_0}$ (because of symmetry, $\vec{E}(r,\theta,\varphi) = \vec{E}(r)$)

$\Leftrightarrow E\int_S dS = \dfrac{Q}{\varepsilon_0}$ $\Leftrightarrow E\, 4\pi r^2 = \dfrac{Q}{\varepsilon_0}$ $\Leftrightarrow \boxed{\vec{E}(\vec{r}) = \dfrac{Q}{4\pi r^2 \varepsilon_0}\, \vec{e}_r}$

2) $\underline{\text{out:}}$ $\underline{r \geq R}$ The total charge is the same, so with the same calcul we have $\boxed{\vec{E}(\vec{r}) = \dfrac{Q}{4\pi r^2 \varepsilon_0}\, \vec{e}_r}$

$\underline{\text{in:}}$ $\underline{r<R}$ $\int_S \vec{E} \cdot d\vec{S} = \dfrac{Q_{int}}{\varepsilon_0}$ $\Leftrightarrow E\, 4\pi r^2 = \dfrac{\rho\, 4\pi r^3}{3\varepsilon_0}$

$\Leftrightarrow \boxed{\vec{E}(\vec{r}) = \dfrac{\rho r}{3\varepsilon_0}\, \vec{e}_r}$ with ρ, the charge density

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3) $\underline{\text{out}}$: Same reasons; $\boxed{\vec{E}(\vec{r}) = \dfrac{Q}{4\pi r^2 \varepsilon_0}\,\vec{e}_r}$
$r \geq R$

$\underline{\text{in}}$: we have $Q(r^n) = \dfrac{4}{3}\pi r^3 \rho(r^n)$ $\quad (n > -3)$
$r < R$

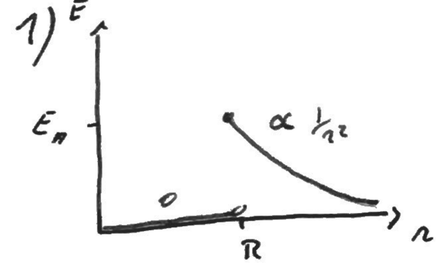
$$= \frac{4}{3}\pi r^3 r^n$$
$$= \frac{4}{3}\pi r^{n+3}$$

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0} \Leftrightarrow E\, 4\pi r^2 = \frac{4}{3\varepsilon_0}\pi r^{n+3} \Leftrightarrow \boxed{\vec{E}(\vec{r}) = \frac{r^{n+1}}{3\varepsilon_0}}$$

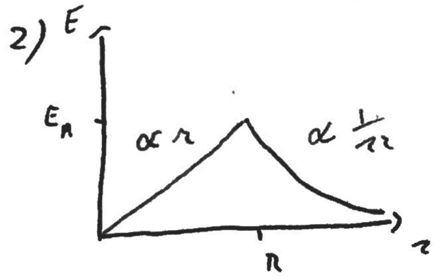
Graphs:

$$\overline{\underline{E_A = \frac{Q}{4\pi R^2 \varepsilon_0}}}$$

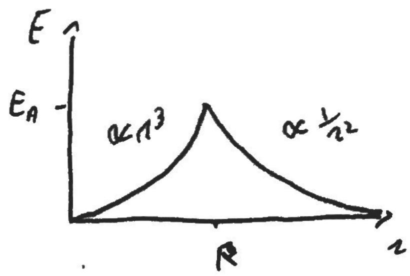
1)



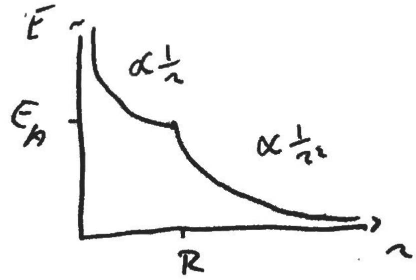
2)



3) $\underline{n = +2}$



$\underline{n = -2}$



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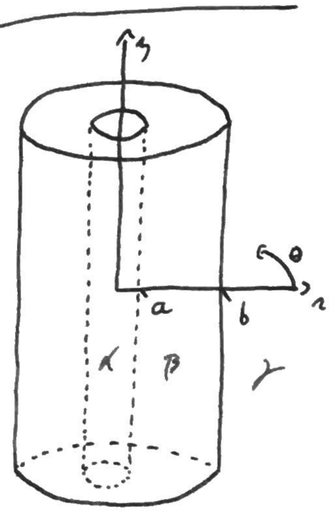
Séance 4 - exo 18:



$\Delta \phi = 0$, by symmetry : $\phi(r,\theta,z) = \phi(r)$

Remember $\Delta \phi(r) = \partial_r^2 \phi + \frac{1}{r} \partial_r \phi \quad (1)$

$$\left| \begin{array}{l} \phi(a) = \phi_a \\ \phi(b) = \phi_b \\ \phi(\infty) = 0 \end{array} \right.$$

we define three regions: α, β & γ

We solve (1) and find $\overline{\underline{\phi(r) = c_1 \ln(r) + c_2}}$

$\underline{\alpha}$: Continuity ; $\phi(a) = \phi_a$

$\phi(0) \neq -\infty$ (the potential can't diverge without charge.)

$\phi(0) \neq -\infty \quad \rightarrow \quad c_1 = 0$

$\phi(a) = \phi_a \quad \rightarrow \quad c_2 = \phi_a$ So $\boxed{\phi(r) = \phi_a}$ $(r \leq a)$

$\underline{\gamma}$: Continuity : $\phi(b) = \phi_b$

$\phi(\infty) = 0$

$\phi(\infty) = 0 \quad \rightarrow c_1 = c_2 = 0$ So $\boxed{\phi(r) = 0}$ $(r \geq b)$

So, everywhere, $\phi_b = 0$

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B: Continuity: $\phi(a) = \phi_a$

$$\phi(b) = \phi_b = 0$$

$$\rightarrow \phi(r) = c_1 \ln(r) + c_2$$

$$\begin{cases} \phi_a = c_1 \ln(a) + c_2 \\ 0 = c_1 \ln(b) + c_2 \end{cases} \iff c_2 = -c_1 \ln(b)$$

$$\iff \begin{cases} \phi_a = c_1 \ln(a) + (-c_1 \ln(b)) \iff c_1 = \phi_a \ln(ab^{-1})^{-1} \\ c_2 = \dfrac{\phi_a}{\ln(ba^{-1})} \ln(b) \end{cases}$$

$$\Rightarrow \boxed{\phi(r) = \frac{\phi_a}{\ln(ab^{-1})} \ln(r) + \frac{\phi_a \ln(b)}{\ln(ba^{-1})}} \quad (a \leq r \leq b)$$

we notice that this equation is continuous with the two others