Paquet d'onde à 1 dim

Note $\displaystyle\int_{-\infty}^{+\infty} d\xi \; e^{-\alpha^2(\xi+\beta)^2} = \frac{\sqrt{\pi}}{\alpha}$ si $\begin{cases} -\frac{\pi}{4} < \arg\alpha < +\frac{\pi}{4} \\ Re(\alpha^2) > 0 \\ \beta \in \mathbb{C} \end{cases}$

à $t=0$: $$\psi(x,0) = \frac{\sqrt{a}}{(2\pi)^{3/4}} \int_{-\infty}^{+\infty} dk \; e^{-\frac{a^2}{4}(k-k_0)^2} e^{ikx}$$

$$= \left(\frac{2}{\pi a^2}\right)^{1/4} e^{ik_0x} e^{-x^2/a^2}$$

$$|\psi(x,0)|^2 = \sqrt{\frac{2}{\pi a^2}} \; e^{-2x^2/a^2}$$

$$\int dx \, |\psi(x,0)|^2 = \sqrt{\frac{2}{\pi a^2}} \cdot \sqrt{\pi}\sqrt{\frac{a^2}{2}} = 1$$

à t : $$\psi(x,t) = \frac{\sqrt{a}}{(2\pi)^{3/4}} \int dk \; e^{-\frac{a^2}{4}(k-k_0)^2} e^{i\left[k x - \frac{\hbar k^2}{2m} t\right]}$$

$$= \left(\frac{2a^2}{\pi}\right)^{1/4} \frac{e^{i\varphi}}{\left(a^4 + \frac{4\hbar^2 t^2}{m^2}\right)^{1/4}} e^{ik_0x} e^{-\frac{\left(x - \frac{\hbar k_0}{m} t\right)^2}{a^2 + \frac{2i\hbar t}{m}}}$$

$$|\psi(x,t)|^2 = \sqrt{\frac{2}{\pi a^2}} \frac{1}{\sqrt{1 + \frac{4\hbar^2 t^2}{m^2 a^2}}} e^{-\frac{2a^2\left(x - \frac{\hbar k_0}{m} t\right)^2}{a^4 + \frac{4\hbar^2 t^2}{m^2}}}$$

$$\begin{cases} \Delta x(t) = \dfrac{a}{2}\sqrt{1 + \dfrac{4\hbar^2 t^2}{m^2 a^4}} = \displaystyle\int dx \, (x-\bar{x})^2 |\psi(x,t)|^2 \\ \Delta p(t) = \dfrac{\hbar}{a} = \displaystyle\int dk \, (p-\bar{p})^2 |\tilde{\psi}(k,t)|^2 \end{cases}$$

Potentiel en escalier

$$\frac{d^2\varphi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\varphi = 0$$

1°) $E > V \longrightarrow \varphi = A e^{ikx} + A' e^{-ikx}$ $\qquad k^2 = \frac{2m}{\hbar^2}(E-V) \geqslant 0$

ondes progressives

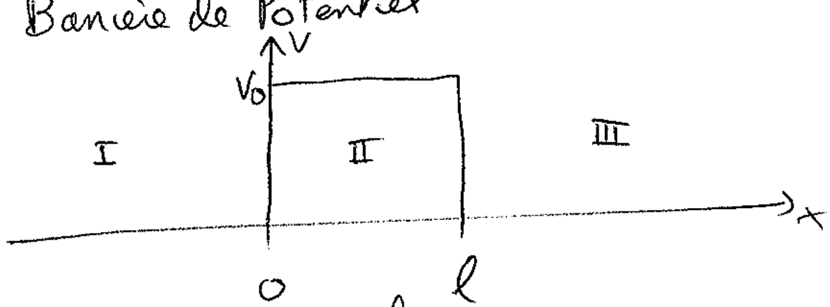
2°) $E < V \longrightarrow \varphi = B e^{\rho x} + B' e^{-\rho x}$ $\qquad \rho^2 = \frac{2m}{\hbar^2}(V-E) \geqslant 0$

effet tunnel.

3°) là où V est discontinu : φ est continu

$\partial_x \varphi$ est continu

Barrière de Potentiel



Onde incidente de la gauche

1°) $E > V_0$

$$\begin{cases} \varphi_I = A_1 e^{ik_1x} + A_1' e^{-ik_1x} \\ \varphi_{II} = A_2 e^{ik_2x} + A_2' e^{-ik_2x} \\ \varphi_{III} = A_3 e^{ik_3x} \end{cases}$$

$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

$k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

Courant en I :
$$\begin{aligned} J &= \frac{\hbar}{m} \operatorname{Im}\left[\bar{\varphi}\, \partial_x \varphi\right] \\ &= \frac{\hbar}{m} \operatorname{Im}\left[(\bar{A}_1 e^{-ik_1x} + \bar{A}_1' e^{ik_1x}) . ik_1 (A_1 e^{ik_1x} - A_1' e^{-ik_1x})\right] \\ &= \frac{\hbar}{m} k_1 (|A_1|^2 - |A_2|^2) \\ &= v_1 (|A_1|^2 - |A_2|^2) \end{aligned}$$

Conditions de raccord

en $x=0$: $\begin{cases} A_1 + A'_1 = A_2 + A'_2 \\ ik_1(A_1 - A'_1) = ik_2(A_2 - A'_2) \end{cases}$

en $x=\ell$: $\begin{cases} A_2 e^{ik_2\ell} + A'_2 e^{-ik_2\ell} = A_3 e^{ik_1\ell} \\ ik_2(A_2 e^{ik_2\ell} - A'_2 e^{-ik_2\ell}) = i k_3 A_3 e^{ik_1\ell} \end{cases}$

$$\longrightarrow \begin{cases} A_2 = \frac{1}{2}\left(1 + \frac{k_3}{k_2}\right) e^{i(k_1-k_2)\ell} \\ A'_2 = \frac{1}{2}\left(1 - \frac{k_3}{k_2}\right) e^{i(k_1+k_2)\ell} \end{cases}$$

$$\begin{cases} A_1 = \frac{1}{2}\left(1 + \frac{k_2}{k_1}\right) A_2 + \frac{1}{2}\left(1 - \frac{k_2}{k_1}\right) A'_2 \\ A'_1 = \frac{1}{2}\left(1 - \frac{k_2}{k_1}\right) A_2 + \frac{1}{2}\left(1 + \frac{k_2}{k_1}\right) A'_2 \end{cases}$$

$$\longrightarrow \begin{cases} A_1 = \left[\cos(k_2\ell) - i\,\frac{k_1^2+k_2^2}{2k_1k_2}\sin(k_2\ell)\right] e^{ik_1\ell} A_3 \\ A'_1 = i\,\frac{k_2^2-k_1^2}{2k_1k_2}\sin(k_2\ell)\, e^{ik_1\ell} A_3 \end{cases}$$

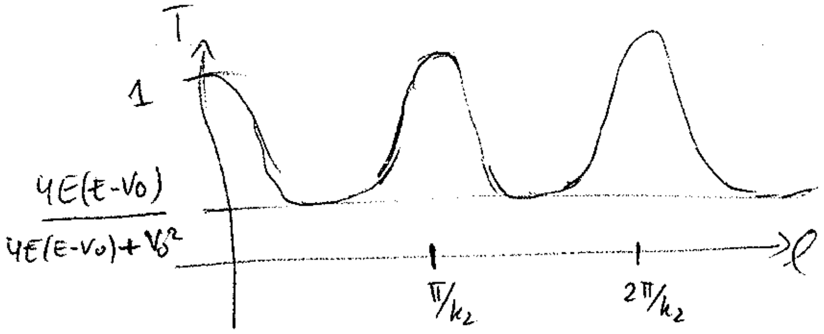
coefficient de réflexion

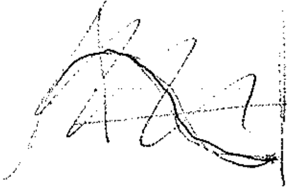
$$R = \frac{|A'_1|^2}{|A_1|^2} = \frac{(k_1^2-k_2^2)\sin^2 k_2\ell}{4k_1^2k_2^2 + (k_1^2-k_2^2)^2\sin^2 k_2\ell}$$

$$T = \frac{|A_3|^2}{|A_1|^2} = \frac{4k_1^2k_2^2}{4k_1^2k_2^2 + (k_1^2-k_2^2)^2\sin^2 k_2\ell}$$

$$R + T = 1$$

$$T = \frac{4E(E-V_0)}{4E(E-V_0) + V_0^2\sin^2\left(\sqrt{2m(E-V_0)}\,\ell/\hbar\right)}$$

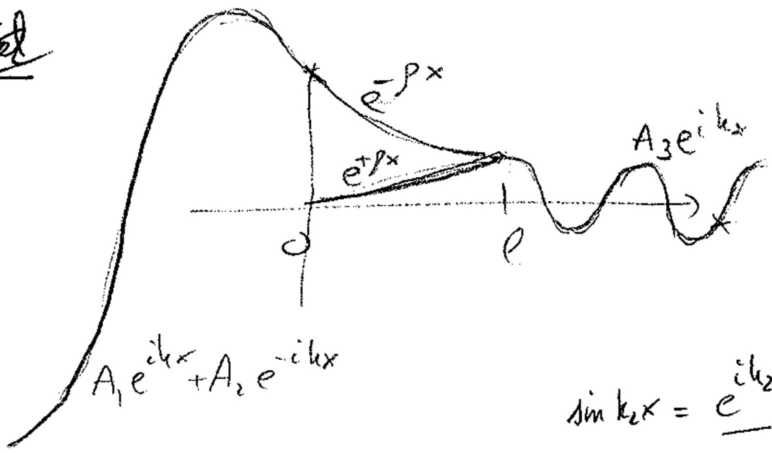




resonances

~~Barrière de potentiel~~

2) $E < V_0$ Effet Tunnel.



$$k_2 \longrightarrow -i\rho_2$$

$$\rho_2 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

$$\sin k_2 x = \frac{e^{ik_2x} - e^{-ik_2x}}{2i}$$

$$\hookrightarrow \frac{e^{\rho_2 x} - e^{-\rho_2 x}}{2i} = \frac{1}{i} sh(\rho_2 x)$$

$$T = \frac{-4k_1^2\rho_2^2}{-4k_1^2\rho_2^2 + (k_1^2+\rho_2^2)^2(-1).sh^2(\rho_2 \ell)}$$

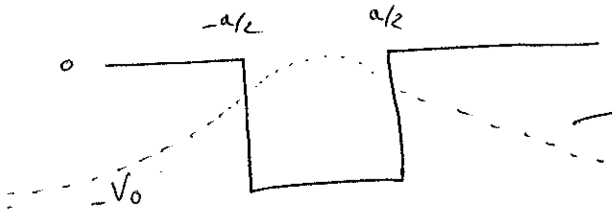
$$= \frac{4k_1^2\rho_2^2}{4k_1^2\rho_2^2 + (k_1^2+\rho_2^2)^2 sh^2 \rho_2 \ell}$$

$$= \frac{4E(V_0-E)}{4E(V_0-E) + V_0^2 \, sh^2(\rho_2 \ell)}$$

$$\simeq \frac{16E(V_0-E)}{V_0^2} e^{-2\rho_2 \ell}.$$

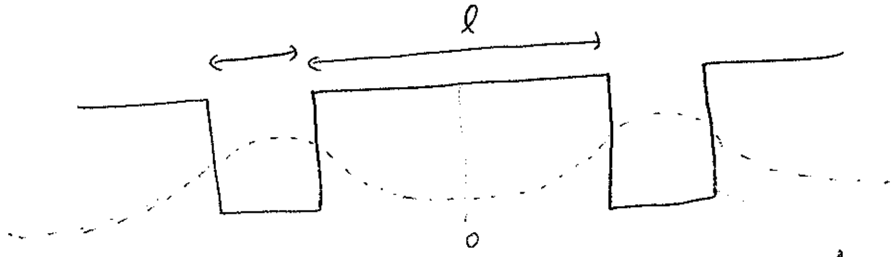
Puit de Potentiel

Cohen p 76



→ solutions paires et impaires

Double puit



→ solutions paires et impaires $\psi(0) = 0$ où $\psi'(0) = 0$

exemple NH_3