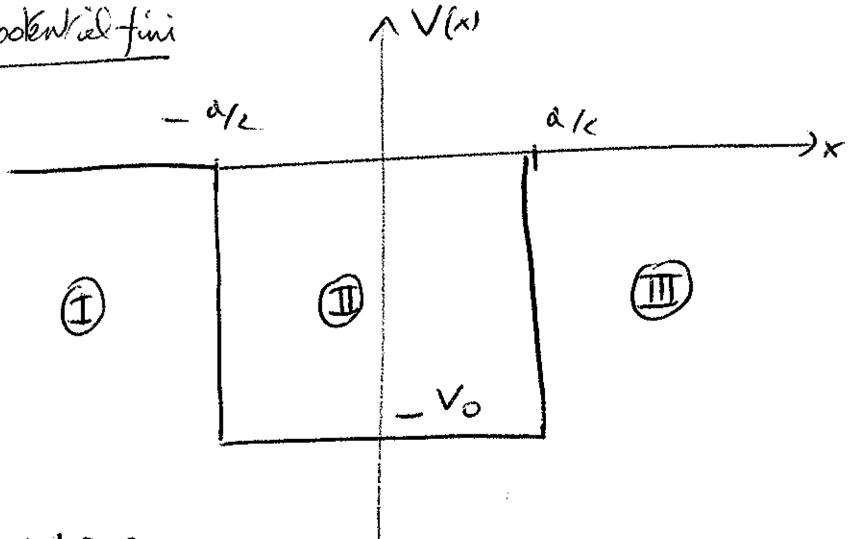
Puits de potentiel fini



$$-\frac{\hbar^2}{2m}\partial_x^2\varphi + V(x)\varphi = E\varphi \qquad -V_0 < E < 0$$

I $\quad \varphi(x) = B e^{+\rho x} \qquad \rho = \sqrt{\frac{-2mE}{\hbar^2}}$

II $\quad \varphi(x) = A e^{ikx} + A' e^{-ikx} \qquad k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$

en $x = -a/2$:

$$B e^{-\rho\frac{a}{2}} = A e^{-ika/2} + A' e^{+ika/2}$$

$$+\rho B e^{-\rho a/2} = ik\left(A e^{-ika/2} - A' e^{+ika/2}\right)$$

$$A = \frac{\rho + ik}{2ik} e^{-(\rho - ik)\frac{a}{2}} B$$

$$A' = -\frac{\rho - ik}{2ik} e^{-(\rho + ik)\frac{a}{2}} B$$

en III $\quad \varphi(x) = C e^{\rho x} + C' e^{-\rho x} \qquad$ et $C = 0$

$$\begin{cases} C e^{\rho\frac{a}{2}} + C' e^{-\rho a/2} = A e^{ika/2} + A' e^{-ika/2} \\ \rho\left(C e^{\rho a/2} - C' e^{-\rho a/2}\right) = ik\left(A e^{ika/2} - A' e^{-ika/2}\right) \end{cases}$$

$$\begin{pmatrix} A e^{ika/2} & e^{-ika/2} & e^{-\rho a/2} & 0 \\ ik e^{ika/2} & -ik e^{-ika/2} & \rho e^{-\rho a/2} & 0 \\ & & 0 & \\ & & 0 & \end{pmatrix}$$

B

15

$$2\rho e^{\rho a/2} C = (\rho+ik)Ae^{ika/2} + (\rho-ik)A'e^{-ika/2}$$

$$= \left[\frac{(\rho+ik)^2}{2ik} e^{-\rho\frac{a}{2}} e^{ika} - \frac{(\rho-ik)^2}{2ik} e^{-\rho\frac{a}{2}} e^{-ika}\right] B$$

$$\cancel{= \frac{e^{-\rho\frac{a}{2}}}{2ik}\left[(\rho^2+2ik\rho-k^2)e^{ika} - (\rho^2-2ik\rho-k^2)e^{-ika}\right]B}$$

$$= 0$$

$$\longrightarrow (\rho+ik)^2 e^{ika} = (\rho-ik)^2 e^{-ika}$$

$$\longrightarrow \frac{(\rho-ik)^2}{(\rho+ik)^2} = e^{2ika}$$

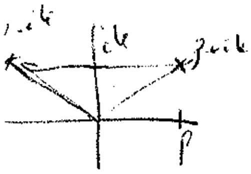
$\longrightarrow$ 1°) $\dfrac{\rho-ik}{\rho+ik} = -e^{ika}$

complexe de phase $\pi + ka$

complexe de norme 1 de phase $-2\,tg^{-1}\left(\frac{k}{\rho}\right)$

2°) $\dfrac{\rho-ik}{\rho+ik} = e^{ika}$

complexe de norme 1 de phase $-2\,tg^{-1}\left(\frac{k}{\rho}\right)$



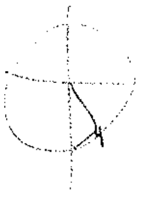
$$\longrightarrow tg^{-1}\left(\frac{k}{\rho}\right) = -\frac{\pi}{2} + \frac{ka}{2}$$

$$\longrightarrow \frac{k}{\rho} = tg\left(-\frac{\pi}{2} + \frac{ka}{2}\right)$$

$$= -tg\left(\frac{\pi}{2} - \frac{ka}{2}\right)$$

$$= \cot g\, \frac{ka}{2}$$

$$= \frac{1}{tg\, ka/2}$$



$$tg\,\frac{ka}{2} = \frac{\rho}{k}$$

$$tg^{-1}\left(\frac{k}{\rho}\right) = -\frac{ka}{2}$$

$$tg\,\frac{ka}{2} = -\frac{k}{\rho}$$

$$\sin^2\frac{ka}{2} = \frac{tg^2\, ka/2}{1+tg^2\, ka/2} = \frac{k^2}{k_0^2}$$

$k_0 = \sqrt{\frac{2mV_0}{\hbar^2}} = \sqrt{k^2+\rho^2}$ indép. de E

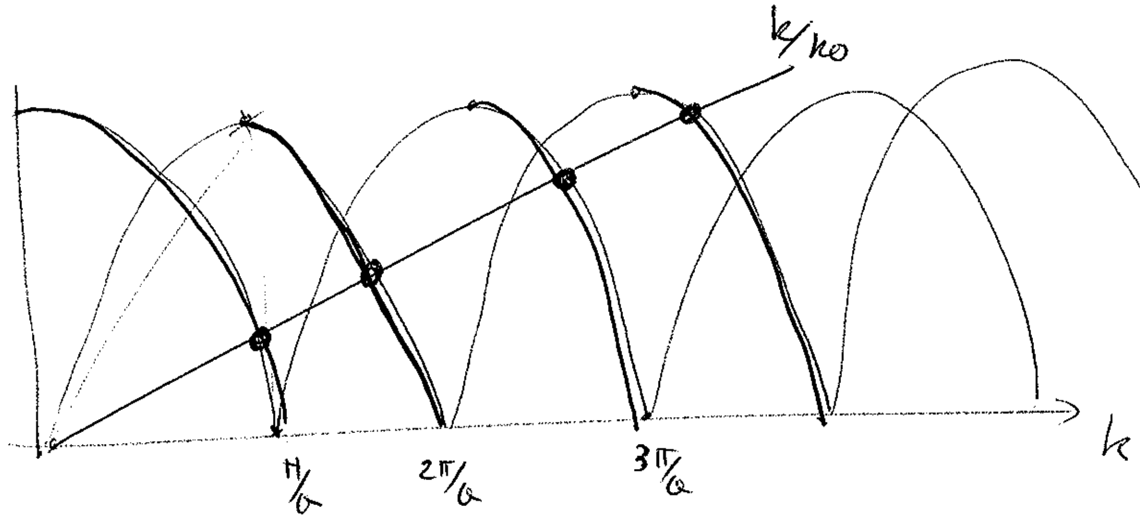
(8)

(16)

$$\frac{1}{\cos^2(ka/2)} = 1 + tg^2\frac{ka}{2} = \frac{k^2+\rho^2}{k^2} = \frac{k_0^2}{k^2}$$

1°) $\Leftrightarrow \begin{cases} \left|\cos\frac{ka}{2}\right| = \frac{k}{k_0} \\ tg\left(\frac{ka}{2}\right) > 0 \end{cases}$

2°) $\Leftrightarrow \begin{cases} \left|\sin\frac{ka}{2}\right| = \frac{k}{k_0} \\ tg\,\frac{ka}{2} < 0 \end{cases}$



si $k_0 \leq \pi/a$

$\Leftrightarrow V_0 \leq V_1 = \frac{\pi^2\hbar^2}{2ma^2} \longrightarrow$ 1 état lié.

$\frac{\pi}{a} \leq k_0 \leq 2\pi/a \qquad \frac{\pi^2\hbar^2}{2ma^2} \leq V_0 \leq 4\frac{\pi^2\hbar^2}{2ma^2}$ 2 états liés.

etc...

si $V_0 \gg V_1$ et les états d'énergie les + bas sont

$\simeq k = n\pi/a \qquad E \simeq \frac{\pi^2\hbar^2 n^2}{2ma^2} - V_0$

Approximation WKB / Semiclassique

$$\left[-\frac{\hbar^2}{2m}\partial_x^2\psi + V(x)\psi = E\psi\right]$$

$$k(x) = \frac{1}{\hbar}\sqrt{2m(E-V(x))}$$

$$\lambda(x) = \frac{1}{k(x)}$$

$$\frac{d\lambda}{dx} \ll 1$$

$$\psi = A(x)e^{i\cancel{S'(x)}/\hbar}$$

$$\partial_x\psi = A'e^{iS/\hbar} + iA\frac{S'}{\hbar}e^{iS/\hbar}$$

$$\partial_x^2\psi = A''e^{iS/\hbar} + 2iA'\frac{S'}{\hbar}e^{iS} + iA\frac{S''}{\hbar}e^{iS} - A\frac{S'^2}{\hbar^2}e^{iS}$$

$$\longrightarrow 2A'S' + AS'' = 0 \longrightarrow \boxed{A = \frac{1}{\sqrt{S'}}}$$

$2(\ln A)' + (\ln S')' = 0$

$$\left(\frac{S'^2}{2m} - \cancel{\hbar^2\frac{A''}{A}}\right) + V(x) = E$$

équation d'Hamilton-Jacobi

$$S'(x) = \pm p(x)$$

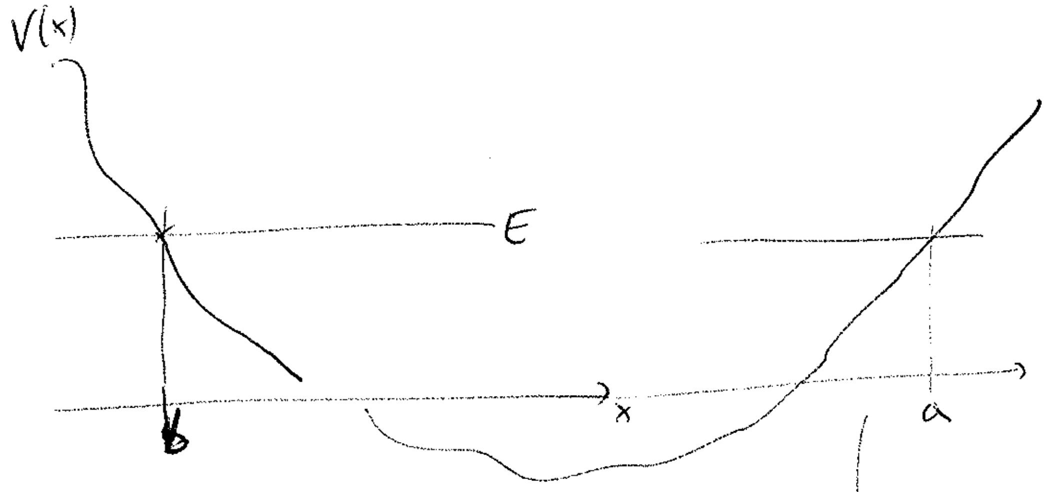
$$S(x) = \pm\int^x dx'\, p(x')$$

$$\psi(x) = \frac{1}{\sqrt{p(x)}}\, e^{\pm i\int^x dx'\frac{p(x')}{\hbar}}$$

– dans une région classiquement interdite.

$$\psi(x) = \frac{1}{\sqrt{\rho(x)}}\, e^{\pm \int^x dx' \rho(x)} \qquad \frac{\rho^2(x)}{2m} = V(x) - E$$

– condition de raccordement.



$$\psi(x) = \frac{1}{\sqrt{k(x)}} \cos\left(\int_b^x k(x')\,dx' - \pi/4\right) \qquad \psi(x) = \frac{1}{\sqrt{k(x)}} \cos\left(\int_x^a k(x')\,dx' + \pi/4\right)$$

→ condition de quantification

$$\frac{1}{\hbar}\int_b^a dx \sqrt{2m(E-V(x))} = \left(n+\frac{1}{2}\right)\pi$$

vitesse de groupe $\dfrac{\partial k(x)}{\partial \omega} = \dfrac{\partial \rho(x)}{\partial E} = v_{cl}(x)$